Invasion Percolation of Fluids into Active Faults using a Cellular Automaton Model

Jianhong Xu

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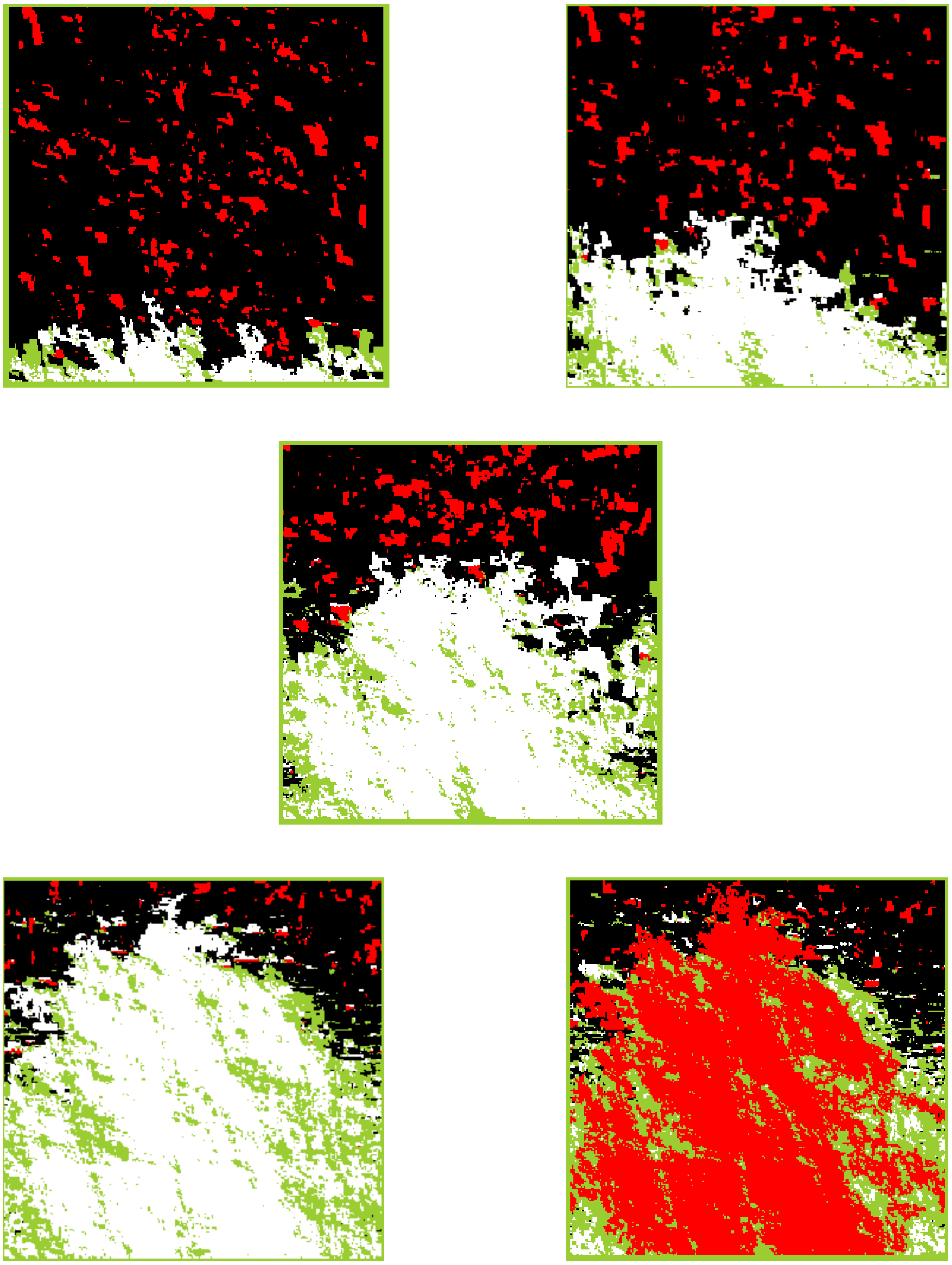
Abstract

One of the desirable goals in seismology is that of earthquake prediction. Lots of research and measurements have been made to observe the earthquake precursors and various models have been produced to simulate the processes of earthquake generation. Many of them suggest these precursors involve the action of fluids. The aim of this project is to generate a model with fluid percolation, to simulate these processes.

This report will first explain the theory behind this model. The exact model will then been presented, followed by the detail of the implementation. Finally the future work on this project are considered.



The Rock System at Different Stages of Evolution

Note: White – wet crack; Red – dry crack; Green – wet healed element; Black – dry non-failed element.

1 Introduction

The forecasting of catastrophe is an ancient and respected occupation. It is only in recent years, however, that earthquake prediction has become a scientifically rigourous pursuit. Due to earthquakes' notorious unpredictability, instead of considering individual earthquakes, more attention has been given to sequences of earthquakes in an area over a period of time.

Recent research shows that the faulting and fluid movement are closely related in real earthquake sequences [Stark and Stark, 1991]. Thus it would seem to be a profitable approach to consider the faults with percolation of fluids when modelling earthquakes.

A two-dimensional model [Maclean, 1992] has been developed on the Connection Machine at the Edinburgh Parallel Computing Centre, which simulates the avalanche-like growth of dry cracks. This project is mainly based on this model while incorporating fluid percolation. The interaction between fluid pressure and fault generation is the main mechanism being investigated here.

In contrast to the model of [Stark and Stark, 1991], pore fluids play an active rather than a passive role in the earthquake generation in this project.

In the following report, Section 2 will run through the theory behind the model. Section 3 and 4 will present the implementation of the model and its result can be found in Section 5. Section 6 contains future work on this project.

2 Seismology

2.1 Patterns and parameters of seismicity

It has been recognised by [Smith, 1986], that certain patterns can be observed in the temporal evolution of parameters which describe the seismicity in an area before a major earthquake. Various parameters have been used to describe the pattern. One of these is the b-value in the Gutenberg-Richter relation:

$$logN = a - bm \tag{1}$$

where N is the number of earthquakes of magnitude greater than m. The b-value describes the scaling property of the seismicity in the area over which earthquakes are observed.

The fractal dimension of the earthquake distribution is another valid seismic scaling parameter, which is weakly negatively correlated with the b-value [Henderson *et al.*, 1991; Hirata, 1989]. This may be determined by box-counting techniques or from earthquake epicentres (correlation dimension).

The b-value and fractal dimension are the two parameters which will be studied throughout the project.

2.2 Rock fracturing and fluid percolation

A fault is a fracture surface along which the rocks on either side have been relatively displaced. The greatest earthquakes are produced by sudden dynamic slips along major geological faults, sometime with offsets of tens of meters taking place in a few seconds.

Various studies show that several major factors contribute to the growth of crack in the rock:

- The magnitude of the externally applied stress
- The size of the crack and the existence of neighbouring cracks
- The presence of fluids

2.2.1 Crack size and interaction

When a crack is introduced into an intact material under compression, the stress distribution in the neighbourhood of the crack is altered. As the crack grows larger, the stress in the region is locally relieved. Thus, as the crack grows bigger, the effective driving stress decreases, the rate of growth decreases, and so a crack will never, on its own, grow beyond a certain size. This results in distributed damage in the material.

On the other hand, when two cracks are close together with each other the stress in the intervening region increases. This makes the crack grow faster, get larger, and hence the stress increases again. This corresponds to the avalanche growth of the crack.

These two effects would work well for the simulation of crack growth on initially intact rock samples in the lab, but our main concern here is much larger faults in the earth's crust for which the avalanche effect is expected to dominate.

2.2.2 Fluid percolation

If a crack exists in a porous, fluid-saturated material, the fluids will percolate into the crack. Once fluid is present, to simulate a tensile structure, effective pressure is increased allowing failure to take place at much lower stress levels in the near future. As a result, it weakens the rock to the point where small earthquakes increase in number and the main shock follows. In my model I examine the effect of this positive feedback between fluid influx and the triggering of earthquake avalanches.

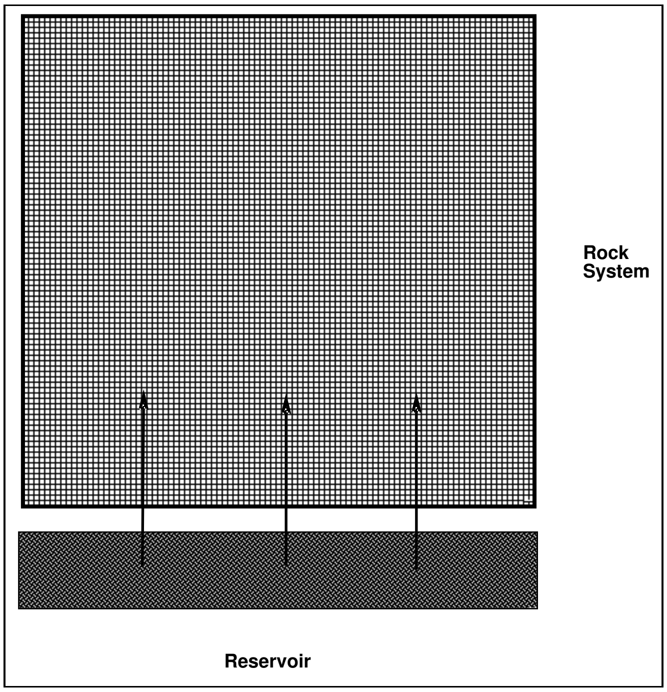


Figure 1: Model outline

3 Model

The model can be visualised as a two-dimensional grid with a reservoir at the base (see Figure 1) which will percolate upwards and influence the earthquake generation.

The main steps of running this model involve:

- setting up the strength for a heterogeneous model "rock"
- cluster cascade if any element fails
- fluid percolation after a cascade

These procedures will iterate until a huge cluster has been generated which connects the grid from top to bottom, which corresponds to a major earthquake. After each iteration, various parameters will be analysed for cluster and fluid distribution.

3.1 Setting up the rock strength distribution

Following Maclean's model, a fractal distribution is used to simulate varying strength in the model "rock". Since rocks exhibit scale-invariance, this is believed to be the most realistic method.

The method used is that of [Huang and Turcotte, 1988]. Each grid element is assigned a random number from a Gaussian distribution. A Fourier transform is applied to the grid, and the resulting coefficients are modified in a consistent manner. A random fractal distribution is then obtained from the inverse Fourier transform, with the fractal dimension determined by the coefficient used in the modification.

The first step is to give each grid element a random value taken from Gaussian distribution. As most random generators generate a uniform distribution, the Gaussian distribution needs to be derived from that. It can be shown that, if a is taken from a uniform distribution $[0, 1)$, and b in $[-\pi, \pi)$, then $c = \sqrt{-2loga} * sinb$ has its value effectively taken from a Gaussian distribution centred about zero.

The next step is to perform a Fourier transform. The Fourier coefficients A_{st} are obtained from the grid values a_{nm} by the following relation:

$$A_{st} = (\frac{1}{N})^2 \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} a_{nm} exp[-\frac{2\pi i}{N}(sn + tm)] \tag{2}$$

After the Fourier coefficients have been obtained, the following operation must be performed on them:

$$A_{st} \Longrightarrow \frac{A_{st}}{(s^2 + t^2)^{\beta/4}} \tag{3}$$

where β is a parameter which defines the resulting dimension, D, by $D = \frac{7-\beta}{2}$.

After performing the inverse Fourier transform, an array with fractal dimension D results. This array is then scaled to make it correspond to the strength of real rocks.

3.2 Crack generation and cluster cascade using cellular automaton

Using a two-dimensional grid to represent the rock system, this model makes use of the cellular automaton "nearest-neighbour" rule to simulate the crack growth, where the "nearest-neighbour" is interpreted as 4 nearest neighbours, see Figure 2(a). This means that each grid element interacts only with the elements around it. However, as evolution proceeds, all grid elements will interact indirectly with all others as the effect propagates throughout the grid.

After setting up the strength for the grid system, the elements which contain fluid need to be recorded. By combining the local stress σ with fluid pressure P, the effective stress for each element is obtained by equation: $\sigma_e = \sigma + P$. Now crack growth proceeds by individual elements "failing". This takes place when the strength of an element is exceeded by its effective stress, i.e. $\sigma_e^{i,j} > \sigma_s^{i,j}$ see Figure 2(a). Once an element has failed, its local stress will be released to its nearest neighbours, which effectively increases the stress on them and causes the cluster cascade, see Figure 2(b) and (c). This procedure will continue until no further crack is generated, i.e. the system reaches a steady state.

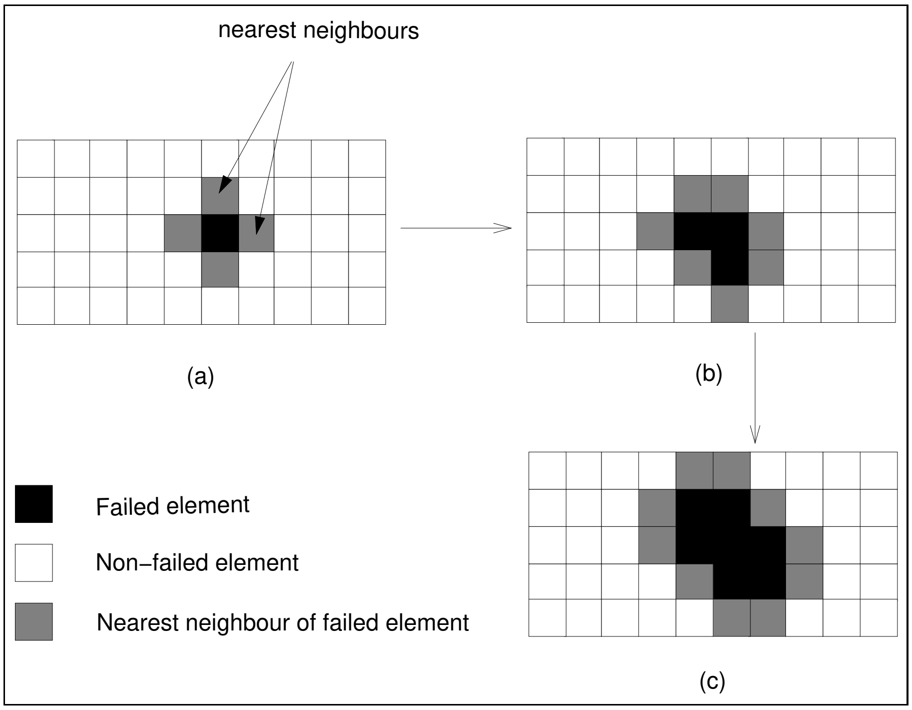


Figure 2: Cluster cascade using cellular automaton

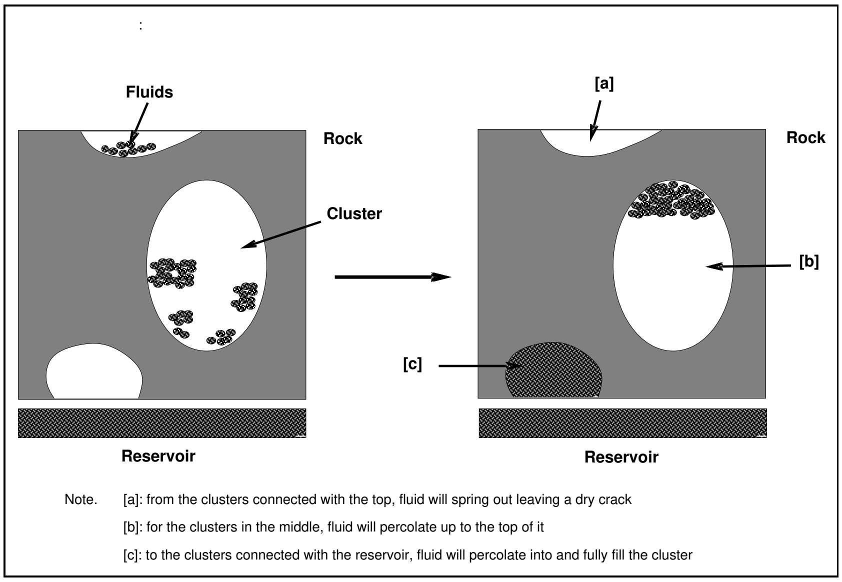


Figure 3: Fluid percolation

3.3 Fluid percolation

Now, if some elements in the cluster are connected to a zone containing fluid, the fluid will be able to percolate upwards.

In the clusters directly connected with the reservoir at the bottom, fluid will percolate in and fully fill the cluster. At those connected with the top, fluid will spring out of the surface and leave a dry cluster. While for the rest, fluid will bubble up to the top of the cluster, see Figure 3. The fluid pressure will then be redistributed throughout the system.

After equilibrating the fluid pressure in this way, the strength of failed elements is restored, to simulate the procedure of fault healing by fluid mineralisation and other mechanisms, and their local stress starts to build up again.

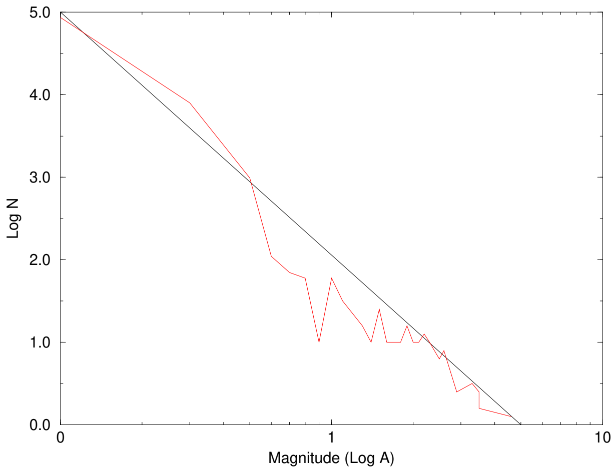
Finally, the remote stress is incremented with the update of the time, and these steps will repeats until a major earthquake is generated.

Here, we assume that the stress incrementation time is longer than the fluid equilibration time and both are much longer than the duration of seismic slip in a single cascade.

3.4 Analysis of crack distribution and fluid emission

After the grid reaches a steady state, analysis needs to be performed on the crack distribution and fluid flux. This includes examination of parameters such as b-value, fractal dimension of the distribution, and the pore pressure at different parts of the fault.

3.4.1 b-value



b-value is a scaling parameter which describes the property of the seismicity in the area where earthquakes are observed. Figure 3.4.1 shows the relationship between the magnitude and frequency of the earthquake. b-value is the approximate gradient shown in the diagram.

To calculate the b-value, the cluster into which individual elements have formed must be found and measurement of its magnitude need to be made. It has been shown [Kanamori and Anderson,1975] that the magnitude of m is related to the area A of the cluster by

$$m \propto logA \tag{4}$$

In this project this relation is used to calculate the magnitude of the cluster.

The b-value, as mentioned above, comes from Gutenberg-Richter relation $logN = a - bm$ and is given by

$$b = \frac{log_{10}e}{\overline{m} - m_0} \tag{5}$$

where $\overline{m}$ is the average magnitude of an ensemble of earthquakes, and m_0 is the threshold magnitude, i.e. the magnitude below which earthquakes are not detected. In this case, m_0 is chosen to be zero, which corresponds to the failure of a single isolated element.

3.4.2 Fractal dimension

The fractal dimension of the crack distribution represents the degree of clustering of seismic activity. It can be calculated by a "box-counting" method, in which the number of "boxes" of a certain size which are needed to cover the cracks is evaluated. A graph of the logarithm of the number of boxes against the logarithm of the box size yields the fractal dimension as its negative gradient.

3.4.3 Fluid flux

Fluid flux can be measured by counting the number of failed elements on the top of the grid which contain fluid inside, and recording the amount of fluid removed from the top layer of the grid.

4 Implementation

This section describes the implementation of the model using CM-Fortran on the Connection Machine (CM).

The CM is a SIMD (Single Instruction Multiple Data) parallel computer. It has a number of processors, each of which holds its own unique data value and executes the same instruction simultaneously. The Connection Machine CM-200 in EPCC has 128*128 processors. Thus a 256*256 array, would be allocated on the CM so that each processors holds 4 array values. When an instruction is passed to be performed on each array element independently, each processor carries out the operation on its own data values simultaneously with the other processors. This is in contrast to the serial approach, in which the operations are performed consecutively in time, and hence take a far longer time for a processor with the same speed.

The CM has one or more "front-ends" associated with it. The front-end processes all the Fortran which does not require the CM—i.e. all serial Fortran. A "controller", which communicates with the front-end and issues array instructions to the individual processors.

Some operations which are essential to the CM include:

- Inter-processor communication. This involves shifting data from one processor to another. It uses NEWS[1] to shift the whole array a certain distance in a particular directions. This is fairly efficient, and is especially useful for models with nearest-neighbours interaction, and in particular to the cellular automaton used in this project.
- Reductions. These involve some sort of reduction of the values held by all the processors into a small number of values. For example, `num=count(A.eq.0)`, counts the number of non-zero element in the array `A` and puts the result into num.

Thinking Machine also provides a software library, the Connection Machine Scientific Software Library (CMSSL). This provides lots of routines for performing operations such as assigning random number to an whole array and performing Fourier transforms, etc.

4.1 The algorithm

This model firstly involves setting up a rock sample with varying strengthes. The iteration can then proceed, according to cluster cascade and fluid percolation rules. Following is the brief algorithm of this model and details of them are discussed in following sections.

[1] North-East-West-South

```
setting up the rock sample

initialise the time and local stress

Iteration start

          increment the time and calculate local stress

          cluster cascade

          cluster distribution analysis

          fluid percolation

          reset the time for failed elements

End iteration if major earthquake occurs
```

Parameter analysis is carried out once the system reaches a steady state. Effectively this is the case following cluster cascade. Fractal dimension and b value can be calculated at this point. Fluid flux and its pressure must be calculated following fluid percolation. Detail about the implementation to calculate these parameters is discussed in Section 4.6.

4.2 Setting up the rock strength distribution

As mentioned in Section 3.1, the rock sample used here is to be assigned strengthes corresponding to a fractal distribution. The main steps include:

- Give the grid a random Gaussian distribution;
- Perform Fourier transform, modify the coefficients, and transform back.

CMSSL provides a routine, `FAST_RNG`, which assigns each of the elements of the prescribed array with a (different) random number between 0.0 and 0.1. Two different arrays are randomised, and combined as described in Section 3.1 to give a random Gaussian distribution.

CMSSL also provides routines for performing Fourier transforms. `FFT` performs either a forward or a reverse Fourier transform as specified. The filtering operation

$$A_{st} \longrightarrow \frac{A_{st}}{(s^2+t^2)^{\beta/4}}$$

is performed on the Fourier coefficients before transforming back.

4.3 Crack generation and cluster cascade

In this model, a time array has been used to store the time for each individual elements in the grid system, and the local stress on each element can be obtained by following relation:

$$\sigma = \mu * \dot{\varepsilon} * t \tag{6}$$

where, σ is the local stress, μ is the shear modulus, $\dot{\varepsilon}$ is the strain rate (assumed constant) and t is the time of the evolution. The increase of the stress then can be achieved by incrementing the time for each element.

By comparing the local stress with the strength using failure criteria mentioned in Section 3.2, failed elements are identified. Now it is needed to find out the number of non-failed neighbours each failed

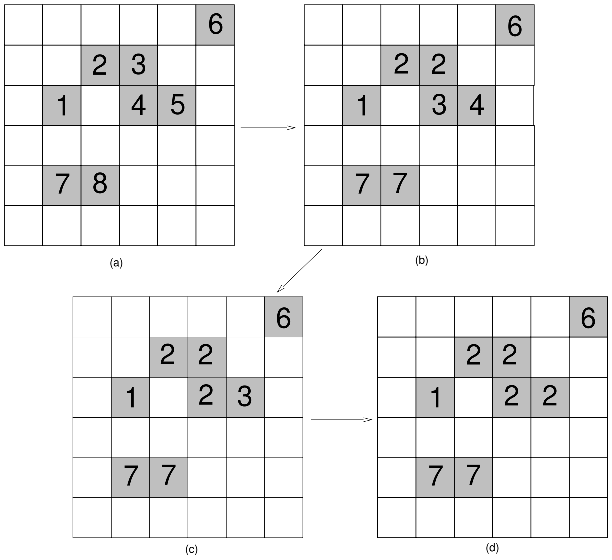


Figure 4: Cluster analysis

element has. To do this, use must be made of the NEWS communication functions `cshift` and `eoshift`. `eoshift` displaces the whole array in top-bottom direction by using static boundary conditions and `cshift` shifts the array along direction of earth's crust using cyclic boundary conditions. Thus two `cshift` and `eoshift` must be performed for 4 nearest-neighbours to find out the number of non-failed neighbours for each failed element.

For failed elements in the middle of the cluster, there are no non-failed neighbours around it. As a result, the local stress is assumed to disappear due to the lack of medium.

For the failed elements at the boundary of the cluster, the local stress is released only to non-failed neighbours. In order to simulate a non-conservative field, only part of its stress is released to each of its non-failed nearest neighbours while the rest is assumed to be dissipated. With the increase of the effective stress, further failure will take place and the cluster will cascade until it reaches a steady state.

To simulate the healing procedure, the stress of failed elements is set to zero by setting the corresponding time to zero. The stress on the failed elements can now be built up again.

4.4 Cluster distribution analysis

A minimum value method is used here to identify clusters. Each failed element is first labelled with a unique value, see Figure 4(a). This is done using an array of the same size as the grid, with non-failed elements labelled to zero. Each element value is simply relabelled with the minimum of its failed neighbours' values iteratively until a steady state is reached. At this point, each cluster is effectively labelled with the minimum of its failed elements' original value. To take the example in Figure 4, the label will change each iteration as shown in Figure 4(b),(c) and (d), finishing up with all elements in the same cluster labelled by the same number. Now different clusters can be easily identified by their unique label.

4.5 Fluid percolation

After identifying the clusters, analysis needs to be carried out for fluid percolation. Three kinds of clusters are distinguished by their positions in the system: clusters connected with the top, clusters connected with the reservoir and all other clusters. For the clusters at the top and connected with the reservoir, fluid percolation is simply achieved by setting the fluid inside the clusters be 0 and 1 respectively.

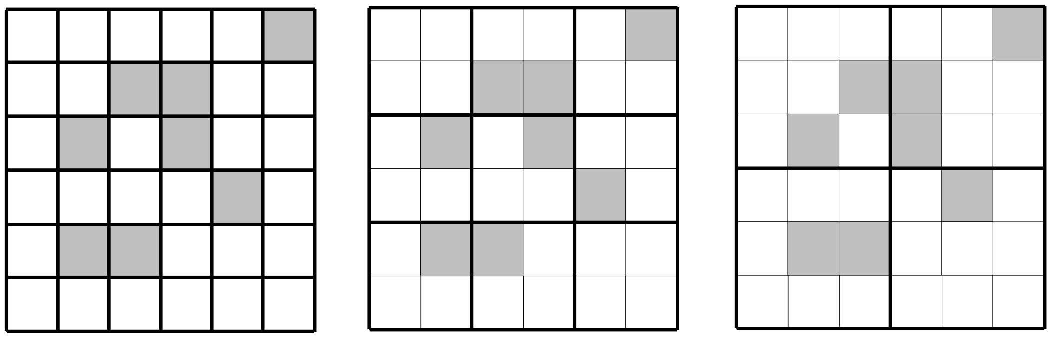


Figure 5: Overlaying boxes in the box-counting method

For the other clusters, those containing fluids are firstly picked out and the "bubble" rule noted in Section 3.3 is applied to them. First, it is needed to calculate the amount of fluids inside the cluster and also the number of failed elements in each line of the cluster. The fluids percolation can then proceed by filling the fluids into the cluster line by line from top to bottom.

4.6 Parameters analysis

Once the rock system reaches a steady state, various parameters need to be analysed. This section describes the exact implementation to calculate these parameters.

4.6.1 b-value

As mentioned in Section 4.4, each cluster was labelled with the minimum of its failed elements' original values after cluster distribution analysis. The magnitude of the cluster is simply determined by the number of times its label appeared in the array. The b value can then be determined by equation 5.

4.6.2 Fractal dimension

The fractal dimension of the cluster distribution is calculated by a "box-counting" method. This involves overlaying the array which holds the crack distribution with square boxes, and counting the number of boxes which contain failed elements. An example is shown in Figure 5, where the black boxes stand for the failed elements.

The graph which results is shown in Figure 6.

It should be noted that grids of side powers of 2 are used in the project, meaning that box sizes of 1,2,4,8,. . . could be used, giving a regular increase in the logarithm of box size.

4.6.3 Fluid flux and pressure

Owing to the method to identify the clusters, those connected with the top can be easily recognised by their unique label. Thus the fluid flux is determined by summing the number of fluids in these clusters.

The fluid pressure is derived by simply counting the number the fluids in the whole rock.

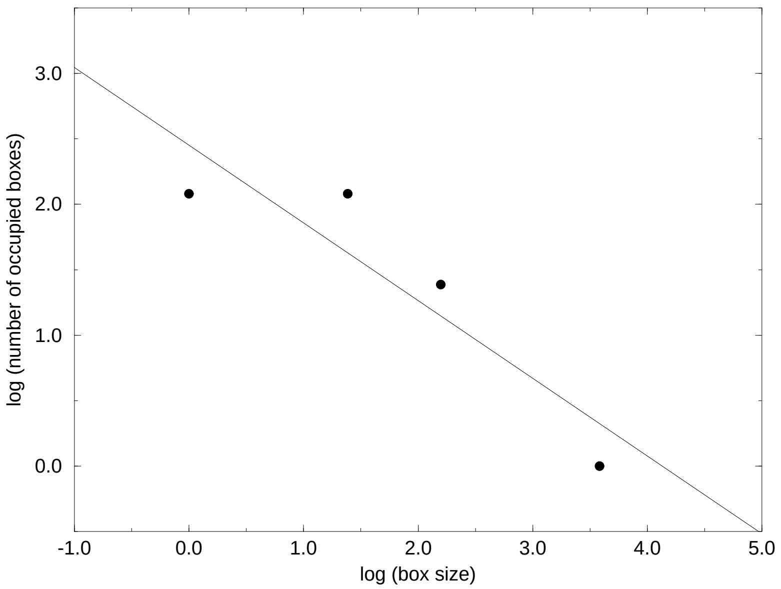


Figure 6: Graph yielding fractal dimension

5 Result from the model

Output of this model via X-window is shown on Page 2. These 5 pictures shows the rock system at different stages of evolution.

The fractal dimension for the strength distribution was set to 2.15 throughout the project, as this is a fairly realistic value.

Figure 7 and 8 shows the results of various parameters from this model under different strength range.

In the low strength condition, the rock is relatively weak. Thus fluids percolate into the rock very easily and dominate the stress for the whole rock. As a result, it weaken the rock furthermore and generate a major earthquake in a short period. While for the rock with high strength, it rarely generate big clusters and hence introduces a barrier for fluid percolation. In this case, local stress is dominant and tends to reach a steady state. So it will take much longer time to produce a big earthquake. Moreover, b value for high strength rock (1.1) is slightly higher than low strength one (1.0).

One thing interesting to notice is that the b value and fractional dimension reach a steady state much earlier than the stress in this model. Due to lack to time, this has not been studied in this project, but it is consistent with models of earthquakes as self-organised critical phenomena.

Frequency of earthquake, fluid gradient and fluid emission have also been calculated in this project and produce the results as expected. The general characteristics of fluid gradient matches very well with real world observation [Sibson, 1994]: the fluid pressure shows a slow increase at the low depth and a sharp increase when it reaches a high level.

These graphs are taken from individual simulations. Even under same strength range, very much different results is likely to be produced. The reason is that the exact strength value at particular positions in grid, rather than strength distribution, decides the possible results from the model, due to the failure criteria used in the model. This highlights the difficulties in predicting short-term fluctuations.

Time performance has been measured for this model. Due to the fact that the results differ from time to time, it is hard to give a precise answer. However, it has been noticed that the procedure of cluster analysis takes around 90

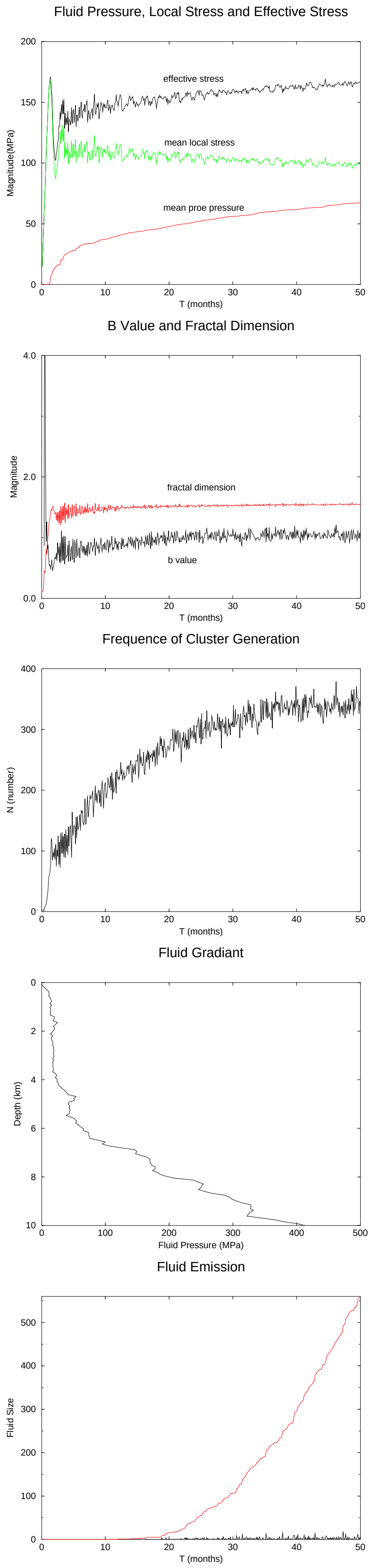


Figure 7: Condition 1: High Strength

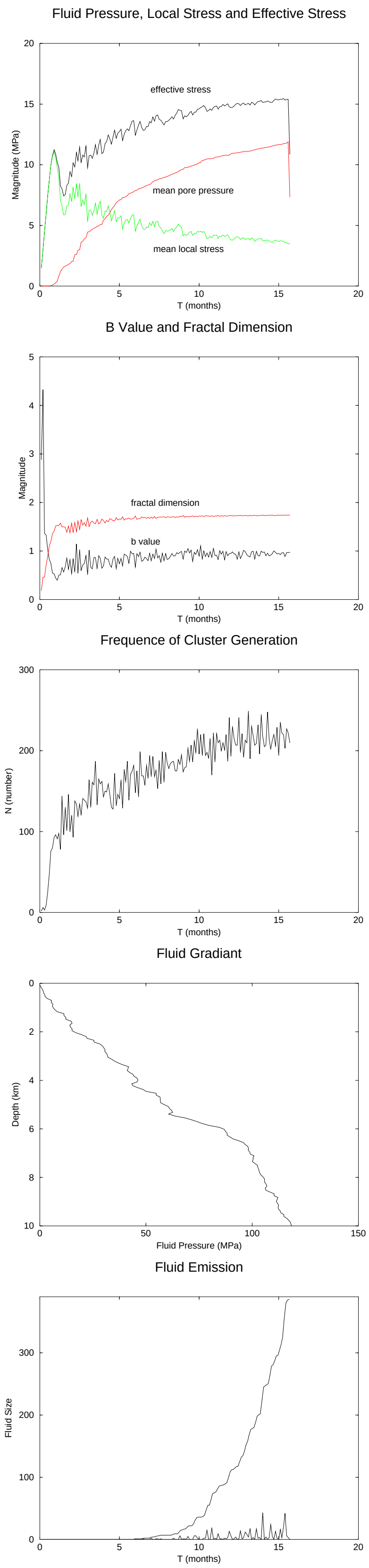


Figure 8: Condition 2: Low Strength

6 Conclusions and further work

As a part of ongoing research being conducted by members of the Geology and Geophysics Department of University of Edinburgh, this project produces an earthquake simulation incorporating both the cluster cascade and fluid percolation for the first time. For generality, all the rules and formula in this model are kept as simple as possible, and some of them may sound a bit unrealistic. However, with this groundwork completed and tested, much more accurate simulation can be fairly easy to achieve, with primarily just minor changes.

This two-dimensional simulation and analysis has been fairly successful in reproducing observed variations in the statistics of earthquakes. Some initial analysis has been done which, while the model is not entirely realistic, has shown that some reasonable results can be achieved. Other factors may be added into this model, such as Mohr Circle failure criteria and residual stress after healing may be added at a later time.

To summarise, this project has successfully reproduce many aspects of fluids trigerring earthquake.

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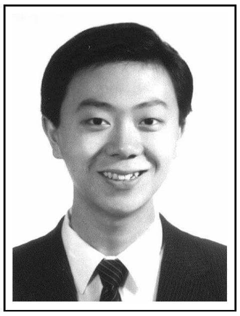

Jianhong Xu is an undergraduate student of Computer Science and Electronics at the University of Edinburgh, UK. This is his first time accessing the parallel computing. As a second year student, Jianhong is interested in Computer Aid Design(CAD) and Computer Communication. Expected to graduate in 1996, he hopes to pursue a career in the above areas.

The project was supervised by Ms Barbara McIness (EPCC), Dr I.G.Main(Department of Geology/Geophysics, University of Edinburgh) and Dr S.A.Wilson (Department of Geology/Geophysics, University of Edinburgh)

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